A SIMPLE, PHYSICAL MODEL OF PARTICULATE WASH-OFF FROM IMPERVIOUS URBAN SURFACES

A Thesis

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ABSTRACT

Particulate matter "washed-off" of impervious surfaces constitutes a large portion of urban nonpoint source pollution. However, current water quality models rely on empirical functions of particulate wash-off that do not meaningfully describe the physical mechanisms involved. In this paper, we investigate the physical mechanisms of rain-flow transportation (Moss et al. 1979) – raindrop induced particle ejection that occurs in shallow flows on moderate slope. Rain-flow transport involves the interaction of both rainfall impact and overland flow, in contrast to the overland flow-dominated, shear-driven particle entrainment that may occur on steep slopes.

We propose a saltation model in which particles are ejected from an impervious surface by raindrop impacts and are translated laterally while settling-out of overland flow. Particles are assumed to be ejected in proportion to rain intensity and the spatial density of particles on the surface. Once ejected, we propose that particles move laterally at the flow velocity and settle according to Stoke's Law. We tested our conceptual model against laboratory flume experiments (10.5 cm wide, 80 cm long) in which rain intensity and upslope overland flow could be independently controlled. The surface of the flume was rough (~1 mm roughness element height) and the particles were 545 μm diameter sand grains. Rainfall rates were between 4.5 and 12.1 cm/hr and overland flow rates were between 150 and 420 mL/min. The conceptual model agreed well with observed data, $R^2 > 0.85$, and was best at the higher overland flows. At low flows the particles spread-out across the surface more than the model predicted. We hypothesize that at low flows lateral movement arising during raindrop impact may be greater than the translation due to overland flow; more research is needed to develop a way to simulate this process. These model results provide a basis

for developing a mechanistic “wash-off” model for spatially distributed urban water quality models.

BIOGRAPHICAL SKETCH

Stephen Shaw was born in Kalamazoo, Michigan on June 13th, 1978. In 1981, his parents moved the young family back to Rochester, NY to run the one vestige of his mother's family's 100-year old dairy business, Weber's Cheese Store. Steve has not escaped a lifelong passion for cheese and other wholesome dairy products.

Steve led an idyllic childhood not marked by any notable traumas. The younger of two brothers, Steve was never without a playmate. Steve and his compatriots' toy-playing years occurred during the golden period of the action-figure; the psyche of the Cold War demanded clear cut differentiation between good and evil, the toy makers had found a profitable synergy between televised cartoons and tangible toys, and video games had yet to become too prevalent. With his collection of action figures, Steve was the last of a generation of youth to rely on their own imagination to create the artifices of their world of play, not using the ready-made contrivances of consumer electronics.

Steve's first scientific success came with his dramatic science fair win in 1991, on the topic of memory, learning and intelligence. Despite this victory, Steve's social life still struggled to bloom. And, alas, it could not overcome the gawky outfits and haircuts selected by an adolescent just breaking from the yoke of a mother's clothes buying tyranny. Steve attended an all-male high school in Rochester, McQuaid Jesuit H.S.

A love of teaching arose from working at a Boy Scout camp in the Finger Lakes, Camp Gorton. In reflecting on his many years there, Steve is still impressed that a camp for 11 to 15 year olds could be run by a group of 16 to 21 year olds. The experience demonstrated to him the potential of dedicated, hard working, and humane organizations. (Steve disavows the current discriminatory policies of the national organization of the

Boy Scouts of America and believes that scouting has been and should remain to be a predominantly asexual experience, a relatively rare occurrence in modern society.)

Without fanfare, Steve decided to attend Cornell University. In the marching band, he learned he could not walk backwards and play an instrument at the same time. Starting in his freshman year, Steve worked in the Soil and Water Lab, the same lab where he currently conducts his research. One of his finest achievements occurred outside the classroom, the crafting of the faux-taxidermy pan fish, Lenny. Steve graduated from Cornell in May of 2000 with a Bachelors of Science degree in the Agricultural and Biological Engineering Department.

After graduation from college, Steve worked for an engineering consulting firm in Northern New Jersey. Steve returned to Cornell in 2003 to undertake his graduate work.

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I would like to thank my committee members - Dr. Tammo Steenhuis, Dr. M. Todd Walter, and Dr. Rebecca Schneider - for their comments and support of my thesis work. Additionally, I would like to extend my thanks to Dr. Jean-Yves Parlange, Dr. Brian Richards, and Dr. Michael Walter for their periodic guidance and encouragement.

Finally, I would like to thank my mother, father, brother and fiancé for their unconditional interest in dirt and the stories I tell about it.

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CHAPTER ONE
INTRODUCTION

Stormwater runoff from urban areas is recognized as a major source of water pollution. In addition to flushing dissolved materials from impervious surfaces, rainfall-runoff events also detach and transport substantial amounts of particulate matter (Sartor et al 1974, Sansalone et al. 1998). Sartor and Boyd (1972) introduced the first particulate "wash-off" model, assuming the rate of particle loss on a catchment scale is directly related to the quantity of particulate available. Current urban stormwater models such as SWMM and HSPF are still based on this original lumped model (Tsishrintzis & Hamid 1998, Bicknell et al. 1997). In addition to being spatially non-specific, models such as SWMM generally rely on extensive calibration of empirical wash-off coefficients (Tsishrintzis & Hamid 1998), a fact that limits their predictive capabilities (Akan 1988). Regulators' recent emphasis on reducing non-point source pollution has increased interest in implementing pollutant management practices in urban areas, a task requiring spatial specificity and predictive abilities. Greater insight into the underlying *physical* mechanisms of particulate detachment and transport could provide a more detailed understanding of the movement of pollutants in the urban landscape.

There are very few published explanations of physical wash-off mechanisms and those are generally borrowed from soil erosion theory, albeit selectively and with limited rigorous, independent experimental validation. One early explanation asserted that the particle loss rate was related to bed shear stresses, although the concept was only supported indirectly, via calibration of a model to plot scale data (Akan, 1988). It is notable that this model neglected the role of rainfall-impact, assuming bed shear induced all particle movement and ignoring the broadly recognized concept of a threshold shear stress, below which overland shear has no effect. Below the critical shear, in studying soil erosion, Moss et al. (1979) first noted the importance of so-called 'rain-flow transportation', a process in which raindrops on shallow water induce particle suspension in a manner similar to turbulent fluctuations in deeper water. Deletic et al. (1997) proposed a wash-off model that included a function to account for these seemingly important rainfall effects and that also recognized the idea of a threshold shear stress calculated from Shield's Curve (Graf, 1971). This model was reasonably well fit to catchment-scale data by employing arbitrary calibration coefficients, but no attempt was made to identify the dominate wash-off mechanisms – rainfall or overland flow - at different times during a storm event or at different catchment locations.

Vaze and Chiew (2003) attempted to directly isolate the respective roles of rainfall and runoff in particle entrainment and transport with plot experiments, but their conclusions were generally descriptive rather than mathematical or mechanistic. In fact, they concluded that rainfall- and runoff-forces were approximately equal in inducing wash-off over a range of rainfall rates and flow depths, a conclusion which seems almost too simplistic.

Because research has not clearly or consistently identified the primary mechanisms behind wash-off on impervious surfaces, especially with respect to rain-impact, this study proposes a simple mechanistic wash-off model and tests it with uncomplicated experiments. This investigation's specific objectives are to: 1) develop a new, physically-based conceptual model to describe particle wash-off from rough, impervious surfaces for conditions in which 'rain-flow transportation' dominates and 2) test this conceptual model with controlled experiments that elucidate the underlying processes. The experiments, by necessity, employ flow conditions for which a critical overland shear stress is not attained in order to illustrate the interaction between overland flow and rainfall in rain-flow transport. Although we do not address the entire range of possible rainfall and overland flow conditions (i.e. conditions at which entrainment by overland flow dominates), we discuss evidence suggesting that rain-impact transport constitutes the dominant process in urban wash-off.

CHAPTER TWO
THEORY

We consider wash-off of a thin layer of particles from a one-dimensional sloping plane. On this plane, particle movement occurs by a saltation-type process that can be conceptually described with a two-layer model in which particles are either at rest on the rough surface or in motion suspended in the shallow flow (Figure 1). Particles enter the shallow flow by raindrop-induced ejection at a rate e (g cm^{-2} sec^{-1}). Particles leave the shallow flow by settling at a rate h (g cm^{-2} sec^{-1}). Before settling out, suspended particles are advected in the overland flow. Particles on the rough surface layer undergo no movement unless hit by a raindrop.

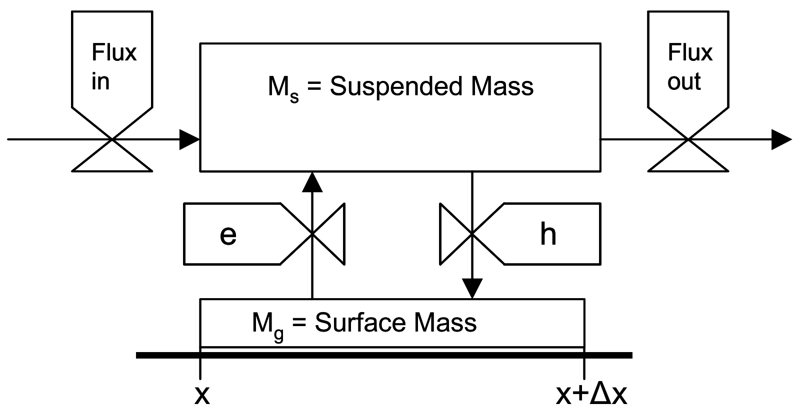

Figure 1. Flow diagram illustrating interaction between particles in suspension and particles on rough, impervious surface for a distinct control volume. Downslope particle motion occurs only when particles are in motion.

One should note that the 'rain-flow transport' modeled here is distinct from rain splash transport, a process we do not consider. In rain splash transport, a particle becomes airborne outside the shallow flow while in rainflow transport the particle always remains within the flow layer. Airborne splash was not considered as the process is multidirectional, not unidirectional downslope, and net transport in any one direction is virtually zero (Moss et al. 1979).

Following Rose and Hairsine (1991) for the case in which flow does not exceed the threshold for entrainment of particles, mass conservation of suspended particles is:

$$\frac{\partial M_s}{\partial t} + \frac{\partial v M_s}{\partial x} = e - h \qquad (1)$$

where M_s is the suspended particle mass (g cm^{-2}), x is the downslope distance, and v is the fluid velocity [cm s^{-1}].

Particle mass on the surface, M_g (g cm^{-2}), at a distinct spatial position is given by:

$$\frac{dM_g}{dt} = h - e \qquad (2)$$

A simple expression for e has been proposed for soil erosion (Hairsine & Rose 1991) and the linear form confirmed by Gao et al. (2003):

$$e = kPH \qquad (3)$$

where H is the unshielded fraction of the bare soil susceptible to rain impact detachment, k (g cm^{-1}) is an experimentally determined constant that accounts for mass loss per drop, and P is the precipitation rate (cm min^{-1}). In soil erosion, a "shield" layer composed of rapidly settling heavy particles develops during rainfall and protects the underlying soil from the impact of raindrops. Thus, for bare soils H is initially one and decreases throughout a rain event (e.g., Heilig at al. 2001). On impervious surfaces, a shield does not form, but availability of transportable particles decreases during a storm event, simply by the fact that a finite quantity of particles is initially present on the surface. Thus, a slightly different expression in which available mass is explicitly noted is proposed: .

$$e = aPM\lambda \tag{4}$$

where a (cm^{-1}) is a rain drop efficiency factor and λ accounts for changes in loss rate due to surface retention. λ will discussed in greater detail in the Results section.

Particle settling rate is:

$$h = \frac{\alpha v_{settle} M_s}{D} \tag{5}$$

where α adjusts bulk concentration to account for variations near the surface, v_{settle} is the particle settling velocity determined by Stokes Law with a drag adjustment factor (Streeter et al. 1988), and D is the depth (cm). Since we are dealing only with very shallow flows and, thus, a minimal vertical concentration gradient in the water column, α is assumed to equal one.

CHAPTER 3
METHODS

Experimental Set-up

Figure 2 illustrates the central elements of our experimental apparatus. A unique attribute of our experimental set-up is that overland flow can be generated separately from rainfall. To distinguish between rainfall-generated runoff and overland flow applied directly at the upslope end of the flume, we will use the terms rainfall-runoff and uplsope flow, respectively. An 80 cm long, 10.5 cm wide stainless steel flume was located beneath a computer-controlled rainmaker (Figure 2). The flume had a 4% slope. A 1 mm roughness surface was constructed of 2 mm glass beads uniformly packed in a single layer and partially submerged in modeling resin (Castin' Craft, ETA Resin Cast Products). A small Plexiglas stilling chamber with an overflow weir was used to control upslope flow. Flow into the stilling chamber was controlled with a variable speed peristaltic pump.

The rainmaker is the same as that used by Gao et al. (2003, 2004, 2005). Four hypodermic needles oscillate along two orthogonal tracks attached to the ceiling of the Soil and Water Lab at the Cornell University Department of Biological and Environmental Engineering, 3 m above the flume. The average raindrop diameter was approximately 1.4 mm. Peristaltic pumps control the rain intensity and the oscillation rates are computer controlled. A full cycle up and down the length of the flume was completed every 28 seconds.

We applied 10 grams (g) of 500-590 μm silica sand particles to the upslope end of the flume surface and then washed-off the particulate with varying combinations of upslope flow and rainfall rates. Although at the larger-sized end of the range of material typically

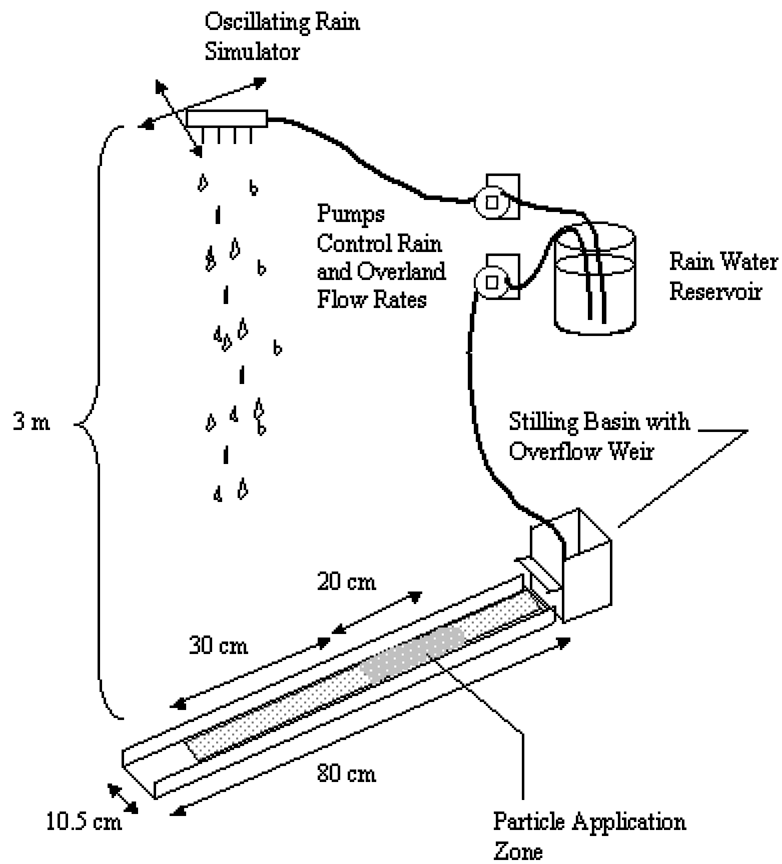

Figure 2. Schematic of experimental set-up and apparatus. Overland flow can be generated independently of rainfall by spilling water from the stilling basin.

washed-off of urban surfaces (Sansalone et al. 1998), 500-590 µm particles were suitable to illustrate typical wash-off processes with our small-scale set-up, i.e., due to the particles rapid settling rate, wash-off rates were low enough to result in meaningful temporal differences in mass loss even with measurements only at one minute intervals. Upslope flow rainfall were systematically varied between 150 and 420 mL min^{-1} (equivalent to a rainfall rate of 0.18 to 0.50 cm min^{-1} applied to the 0.8 m flume length) and 0.08 and 0.20 cm min^{-1} (4.6 and 12.1 cm hr^{-1}), respectively. The direct rainfall rates are comparable to intensities produced from a 2-yr, 15-min storm to a 20-yr, 15 min. storm in the Northeastern US. Prior to each run, the bed of the flume was wetted and uniform flow was established. The particles were pre-wetted and uniformly spread across the 20 cm long application zone located 30 cm from the bottom of the roughness surface (Figure 2). The small fraction of particles sometimes lost during particle application and establishment of the upslope flow were measured in a sample collected during the set-up process. In all experiments, once the initial particle distribution was established there was no discernable particle redistribution prior to starting the rain machine.

Flow off the end of the flume spilled into a stainless steel trough that diverted the sediment-laden water into a funnel fitted with a medium paper filter (Fisher Scientific). Each run was typically 22 minutes long. Filter paper was exchanged every minute for the duration of the run. The filters and particles were air dried, and the sand particles were transferred into a measuring tin and weighed. At the end of each run, the bed was rinsed and any remaining particles were collected, dried, and weighed to close the mass balance of particle loss. The only exception to this procedure was Run 12 where the particle distribution along the slope was determined. After 6 minutes the inflows were stopped. A final rinse was not conducted; instead, particles were collected from five sequential 10 cm intervals. For all runs, on average, >90% of initially applied particles were recovered. The upslope flows, rain intensities, and particle recoveries for all experimental runs are summarized in Table 1.

Table 1. Summary of upslope flow, rainfall rates, and particle recovery for each run.

Run	Upslope Flow (mL min^{-1})	Upslope Flow Equivalent Rainfall Rate (cm min^{-1})	Rainfall (cm min^{-1})	% Particle Recovery
1	150	0.18	0.08	86
2	160	0.19	0.14	92
3	148	0.18	0.18	89
4	320	0.38	0.08	96
5	310	0.37	0.14	86
6	310	0.37	0.15	84
7	305	0.36	0.13	85
8	315	0.38	0.18	96
9	415	0.49	0.08	88
10	420	0.50	0.13	99
11	415	0.49	0.20	94
12*	305	0.36	0.14	99

*Note: Run 12 was stopped after 6 min in order to determine the particle spatial profile on the flume.

[The upslope flow rates is reported in depth per unit time as well as volumetric units to allow direct comparison of rainfall and upslope flow. Reporting upslope flow in depth per unit time is not inherently intuitive but the volumetric equivalent can be conceived as the cumulative flow measured at the end of an 80 cm by 10.5 cm flume resulting from uniform application of the given rainfall rate. However, it must be kept in mind that in the actual experiment, this volumetric flow was constant along the entire length of the flume.]

An empirical relationship between flow and velocity was developed from ten velocity measurements made over a range of overland flow rates applied only as upslope inflow. Flow velocity was determined by measuring the time for dye (FD&C red dye No. 40), injected into the flow stream with a pipette, to travel 50 cm; the measurement was made over the middle section of the flume to avoid end-effects. Velocities measurements were made in triplicate with the coefficient of variation found to be approximately 5%. An average flow depth was determined by continuity. One should note that the calculated depth was sometimes less than the average particle diameter (545 μm) although partially submerged particle were never observed. However, this finding is not unexpected given a characteristic roughness height of approximately 1 mm. Thus, the flow depth is assumed to be indicative of the water height above the roughness elements.

Model Implementation

We used a finite difference model to solve our system of equations. In addition to the sediment mass balance, Eqns. 1 - 5, we also had to account for the water balance. At a point x, the water balance is given by:

$$q\big|_x = PxW + q_o \tag{6}$$

where P is the rain rate per unit width (cm min^{-1}), W is the flume width (cm), q is the flow rate (mL min^{-1}) and q_o (mL min^{-1}) is the constant upslope inflow.

The depth of flow at each point on the flume was determined by continuity and the experimentally derived relationship between q and the flow velocity. The momentum equation with the kinematic assumption was investigated but ultimately rejected because the Manning's roughness value varied with q (Anderson et al. 1998).

While the particle mass balance in Equations 1-5 has been presented in grams per unit area, in implementation we simply track mass. Our experiment also measures total grams, not concentration. The application of the Method of Characteristics to Eq. 1, (Myint-U and Debnath, 1987) implies:

$$\frac{dx}{dt} = v \tag{7}$$

and

$$\frac{dM_s}{dt} = e - h \tag{8}$$

with the initial and boundary conditions: at $t = 0$, $x \geq 0$, $M_s = 0$ and at $t > 0$,

$x = 0$, $M_s = 0$.

Eq. 8 can be written in discrete form as:

$$M_s(x,t) = e(x,t)\Delta t - h(x,t)\Delta t + M_s(x - \Delta x, t - \Delta t) \qquad (9)$$

with particle settling rate given by:

$$h(x,t) = \frac{0.5 v_{settle} M_s(x, t - \Delta t) + 0.5 v_{settle}(x - \Delta x, t - \Delta t)}{D} \qquad (10)$$

and ejection rate given by:

$$e(x,t) = 0.5 aP\lambda M_g(x, t - \Delta t) + 0.5 aP\lambda M_g(x - \Delta x, t - \Delta t) \qquad (11)$$

A simple function for λ is proposed as:

$$\lambda = \left(\frac{M_g}{M^*}\right)^b \quad \text{for } M_g < M^* \qquad (12)$$

where M^* is the mass of particles on the impervious surface at which "effective" coverage of the surface occurs, i.e. roughness elements are sufficiently filled such that the roughness does not diminish the effective impact area of a rain drop. This formulation maintains units of g/min for *e* while establishing a "scaling" factor on *a* between zero and one. The parameter *b* in Eq. 12 is included to generalize the function as there is no *a priori* indication of exactly how *e* should be related to the mass of transportable particles on the surface.

With Eq. 9 substituted into a discrete form of Eq. 2, M_g is updated after each successive new calculation of $M_s(x,t)$:

$$M_g(x,t) = -M_s(x,t) + M_s(x - \Delta x, t - \Delta t) + M_g(x, t - \Delta t) \qquad (13)$$

The explicit finite difference model was run with a time step of 0.00025 min, which was less than the minimum settling time. The spatial step, Δx, was determined by Eq. 7 written in a discrete form:

$$\Delta x = v\Delta t \, . \tag{14}$$

CHAPTER 4
RESULTS

Experimental Observations

Figure 3 summarizes the observed mass loss rates grouped by similar upslope flow rates. Because of inherent sensitivity in the experimental apparatus, it was difficult to precisely duplicate upslope flow rates and rainfall intensities and, thus, "similar" typically indicates <5% differences (see Table 2 for specific rates). For each upslope flow rate, the time to peak and the magnitude of the peak increased with rainfall rate (Figure 3). The magnitude of the wash-off peak also increases with upslope flow (Figures3a-c). Overall, the most rapid breakthrough and highest peaks occur, as expected, with the highest combination of rainfall and upslope flow. Conversely though, low upslope flow not only results in small peaks and delayed breakthrough, but also a change in the shape of the breakthrough curve. At the upslope flow rate of approximately 150 mL/min, the breakthrough curves plateau over a long period of time (Figure 3a). This is in contrast to the sharp peak and long decay of the higher upslope flow rate runs (Figures 3b,c).

We verified experimental repeatability for the conditions of ~300 mL/min upland flow rate and 8.5 cm/hr rainfall rate by running three replicates (Figure 3b, square symbols). In comparing mass loss at each 1 minute interval, the average coefficient of variation was 0.15 while the maximum was 0.77.

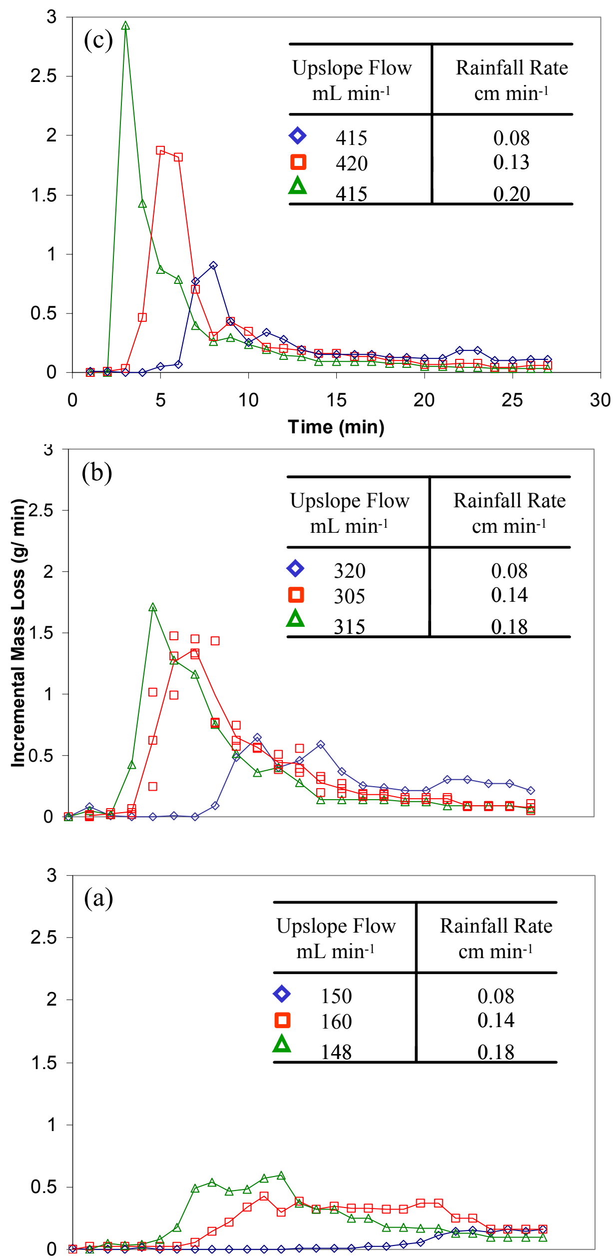

Figure 3. Breakthrough curves observed for runs 1 to 11. Curves are grouped within (a) to (c) by similar upslope flow rates. The rainfall rate (P) (cm min^{-1}) and upslope flow (mL min^{-1}) rate are indicated, respectively. Lines are included to help follow individual experiments. The (305 mL min^{-1} / 0.14 cm min^{-1}) Run was done in triplicate to indicate repeatability of the experiment; data points from all three runs are shown.

Model Results

The model was fit to nine experimental runs and the model parameters *a* and *b* were determined for each run; note, replicated runs 5 – 7 (Table 1) were modeled as a single run using measurement averages. Figure 4 shows examples of model agreement with experimental data. The best fit *a* parameters and R^2 of each run are summarized in Table 2. Values of *a* varied without any clear relationship to upslope flow or rainfall but on average had a value of 130. As a group, *a* values associated with the 150 mL/min series were lowest although these were adjusted to maximize the R^2 and not to replicate the breakthrough curve shape, resulting in breakthrough occurring slightly later than observed.

Table 2. Summary of parameters used to fit numerical model to observed data. The (305 / 0.14) Run is fit to the average of the three replicate experimental runs, numbers 5-7. In all cases, *b*=1.

Upslope Flow ($mL\ min^{-1}$)	Rainfall ($cm\ min^{-1}$)	*a* (cm^{-1})	R^2
150	0.08	95	0.90
160	0.14	105	0.72
148	0.18	115	0.78
320	0.08	145	0.85
305	0.14	160	0.91
315	0.18	145	0.95
415	0.08	130	0.89
420	0.13	130	0.90
415	0.20	140	0.94

Unlike *a* which was used as a calibration factor, the parameter *b* was only adjusted to establish the general form of the function λ . With *b* equal to zero, wash-off is simply proportional to the amount of mass on the surface and occurs as if the particle pulse moved by plug flow with minor dispersion. With *b* less than zero, the wash-off rate is disproportionately larger than particle mass on the surface while with *b* greater than zero the wash-off rate is disproportionately smaller than particle mass on the surface. As would be expected, with $b = -1$, breakthrough occurred too rapidly with too pronounced a peak (not shown). In contrast, the parameter $b = 1$ provided an acceptable fit to the observed breakthrough curves (Figure 5). With b equal to 2, the observed break-through shape is replicated but the model underestimates peak loss (Figure 5). In assessing the response of the model to changes in b shown in Figure 5, a was adjusted as to achieve a best fit. For b=2, a was 235 and for b=0, a was 110.

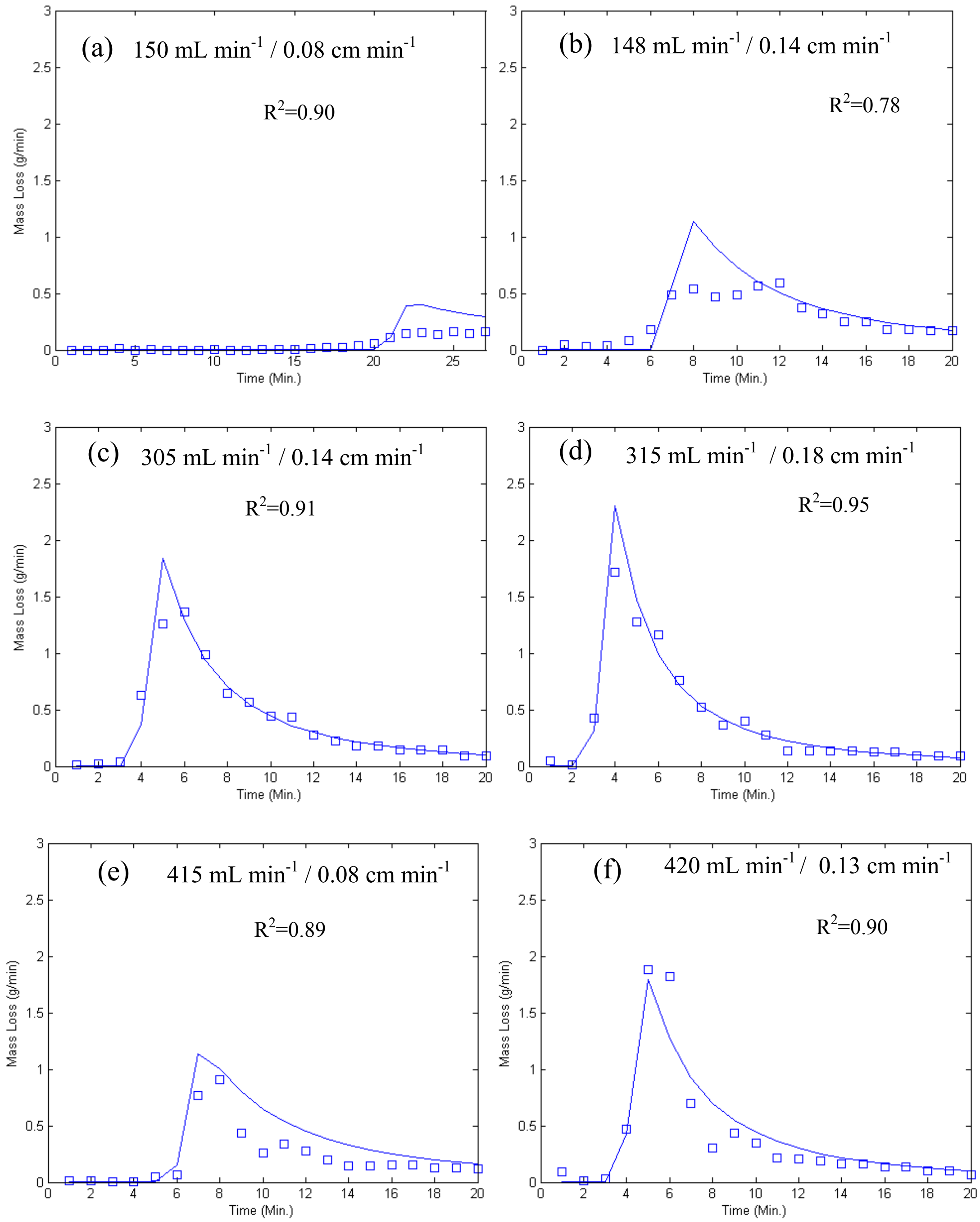

Figure 4. Comparison of selected experimental data with the model. The upslope flow rate and rainfall rate of each run are shown in each figure, respectively. The model fits the general shape of the experimental data relatively well in nearly all cases although dispersion across the surface during low upslope flows cannot be replicated.

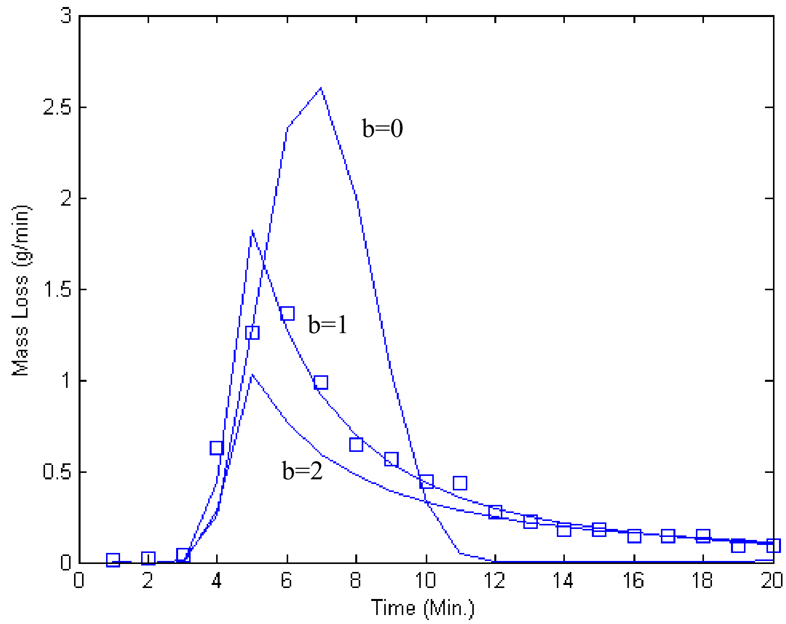

Figure 5. The sensitivity of the model to the selection of the parameter *b* is assessed for the 305 mL min^{-1} / 0.14 cm min^{-1} Run. With *b*=0, mass loss occurs nearly as if the particle pulse moved as plug flow. With *b*=2, the peak is underestimated, even when *a* is adjusted. With *b*=1, the model closely fits the observed data.

As observed in Figure 4, the model does not closely replicate the observed shape of the low upslope flow runs (~ 150 mL min^{-1}). The model can reproduce the rapid peaking and gradual decay exhibited by high flow runs, but does not reproduce the prolonged, relatively constant rate of particle loss for the lower flow runs.

CHAPTER 5
DISCUSSION AND CONCLUSIONS

Unlike previous wash-off models (Akan 1988, Deletic et al. 1997), our model incorporates a function for particle settling. In the other model formulations, once a particle was entrained, it would leave the system, similar to the anticipated behavior of a dissolved substance. In reality, the flow and the rain interact in more complicated ways to control rain-flow transport. The flow controls how far an ejected particle moves with deeper flows and/or faster flows increasing particle "jump" distances by increasing settling time and lateral saltation, respectively. Thus, models that do not account for particle settling do not capture the fundamental rain-induced transport processes.

Additionally, the results suggest that mass loss rate is dependent on the square of the available particulate material once below a critical particle spatial density. Physically, this suggests that as available particles become increasingly scarce on the surface, efficiency of transport decreases disproportionately. As one consequence, the model predicts that particles will be retained on the surface long after the bulk mass front has passed. We ran one experiment (Run 12, table 1) for a short period (6 min) and sequentially rinsed the flume surface to determine how particles were distributed during a wash-off event. Figure 6 shows the measured distribution of the particles on the flume surface for Run 12 and predicted distributions for a series of time periods (Figure 6). The model captured the general distribution of particles at t = 6 min.

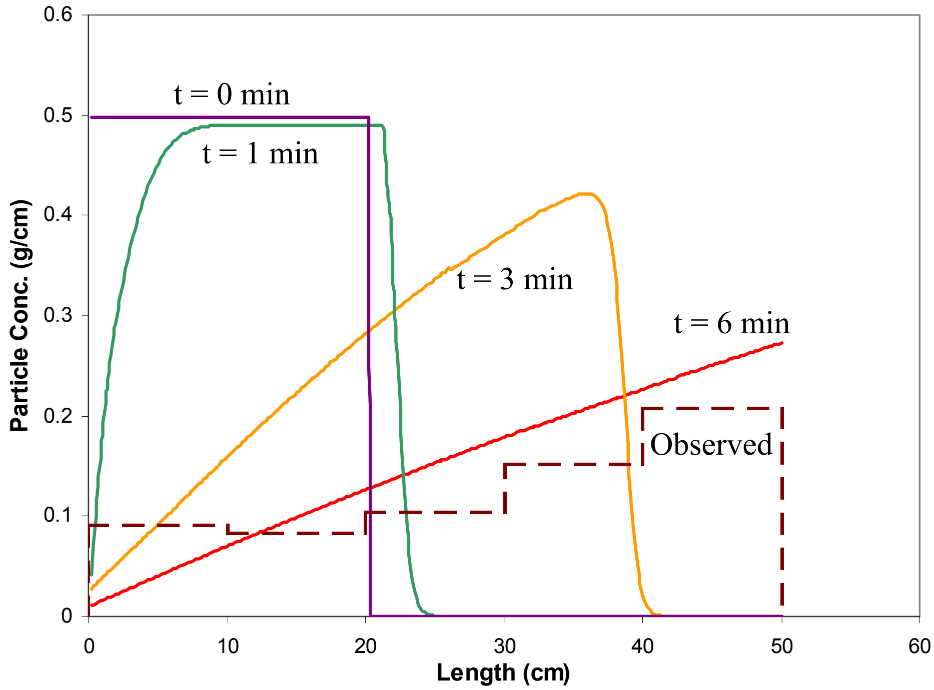

Figure 6. Particulate distribution profiles at different times as predicted by the model for a run with P of 0.14 cm min^{-1} of and upslope flow of 305 mL min^{-1}. Also shown is the observed particle distribution after 6 minutes. The presence of particulate across the full length of the flume suggests that the roughness surface traps and attenuates a sizable fraction of particulate.

Determining a from Fundamentals

For implementation of the model to realistic situations, it is useful to derive the parameter *a* on physical grounds. Lisle et al. (1998) conceptualized particle movement as a two-state Markov process, i.e., the particle is either in a state of motion or a state of rest and is unaffected by its previous history of movement. Lisle et al. (1998) proposed a physical meaning for the probability of being in motion per unit time and arrived at an expression analogous to *a* in Eq. 2:

$$a \approx \frac{A_0}{V} \tag{15}$$

where A_0 (cm^2) is the impact area of a single rain drop and V(cm^3) is the drop volume.

The average a value in our study is ~130, implying an A_0 radius approximately 3.5 times the average drop radius of 0.07 cm. Previous studies have measured the radius of the subsurface cavity generated by a drop falling into a shallow liquid, which seems like a reasonable means to estimate A_o. For a 0.07 cm radius drop falling near terminal velocity (similar to our experiments), cavity radii were found to be between four (Macklin and Metaxas 1975) and seven (Prosperetti and Oguz 1993) times the drop radius. Our result of 3.5 is closer to the findings of Macklin and Metaxas, not unexpected as their work was carried out on thin flows where the surface boundary would constrain development of the cavity. The variability in a among experiments may be due to different flow depths as well as limitations in experimental measurement precision (i.e. measurements at 1 min. intervals due to oscillating nature of rain machine).

Determination of M^*

M^* was introduced as the particle spatial density at which roughness elements are sufficiently filled so that the surface roughness does not impede movement of the upper-most layer of particles when within the effective impact radius of a raindrop .

Assuming that 545 μm diameter spherical particles are uniformly packed in a single layer on the surface, the spatial density would be 0.18 g cm^{-2}. M^* would presumably occur below this point as roughness elements only need to be filled; complete surface coverage is not necessary. Within the model, an M^* of 0.05 g cm^{-2} was used, given that approximately 25% of the total surface area is void space between the uniformly packed 1 mm glass beads.. This also corresponded to the initial application spatial density used in the experiments, a point at which particles appeared to be distributed throughout all voids of the roughness elements.

Note that M^* and a are multiplicative factors in the model and incorrect estimation of M^* can be compensated for by a. Underestimating M^* will necessitate increasing a (although M^* and a are directly proportional, M^* more critically acts as the threshold at which M_g is scaled by M^*). Conversely, overestimating M^* will require decreasing a to fit data to the model. Thus, given the apparent physical reasonability of a, M^* is also probably within a realistic range. Additional experimentation is needed to verify a physically meaningful M^* as well as to investigate how M^* will change with particle size and surface roughness.

Applicability of Model to Low Flow Conditions

The model appeared to breakdown for low overland flows conditions (Figure 4a), suggesting that transport processes different than those accounted for in the model and observed in the higher flow experiments may dominate. In particular, plateaus are apparent in the breakthrough curve of Runs 1 – 3 (Figure 4a), which suggests that some process is uniformly spreading-out the particles. Our model consistently produces a gradual decay following a rapid peak in mass loss. We do not think the behavior shown in figure 4a is the result of enhanced dispersion in front of the mass pulse because the time of breakthrough is not appropriately shorter. One possible explanation is that substantial backward particle movement may occur during raindrop-impact. This may occur if occasional upstream-directed momentum transferred to the particle from a raindrop is greater than the downstream-directed momentum of the bulk flume flow. Therefore, logically, upstream momentum would only be likely to be greater than downstream momentum in the lowest overland flow runs. While the Froude number would not be expected to be an exact proxy for upslope particle movement, it was found that only the 150 mL min^{-1} flow series had Froude numbers less than 1, indicating subcritical flow. Additional investigation needs to be done to ascertain the exact process by which particle transport occurs at lower flows.

Fraction of Total Wash-off Due to Rain-Transport

It is unclear how rain-flow transport and shear-induced transport interact when both processes are present, although, literature suggests that overland shear stress supercedes rain-flow transport as flow depths and/or slopes become large (Moss er al. 1979). However, even in large intensity events, one would expect overland-shear to become important only on the downslope end of longer reaches due to flow accumulation. Thus, even considering the accepted dogma, rain-flow transport would be always play a role in some parts of the landscape. By way of example, we used the rainfall intensity data presented by Vaze and Chiew (2003) in their recent wash-off study and determined the accumulated flow length and storm return periods (Melbourne, Australia) at which overland shear forces exceed the thresholds given by the Shield's diagram (Figure 7). Even for frequent storms, the reach length would need to exceed approximately 4 meters for 100 μm particles to be moved (Figure 7). In a typical urban environment consisting of roadways, sidewalks, and roof surfaces, one would expect that the typical continuous reach length would be on the order of 10 m, leaving a sizable fraction of surfaces unaffected by overland shear forces, suggesting observed wash off in these areas is due to rain-flow transport.

We also ran some simple supplementary experiments with our experimental set-up to investigate the role of roughness on the critical shear stress. While the Shield's Curve has been used to estimate the critical shear stress on impervious surfaces, it was originally developed from data in channels with loose beds. We applied upslope flow (with no rain) at rates for which the shear should have exceeded Shield's critical shear; Specifically, we used a 1600 mL min^{-1} flow and observed minimal particle loss even though the Shield's Curve predicts a critical shear threshold near 1100 mL min^{-1} flow. We suspect that surface roughness may increase the critical shear stress, thus further increasing the fraction of the urban landscape not necessarily subject to wash-off from overland shear forces.

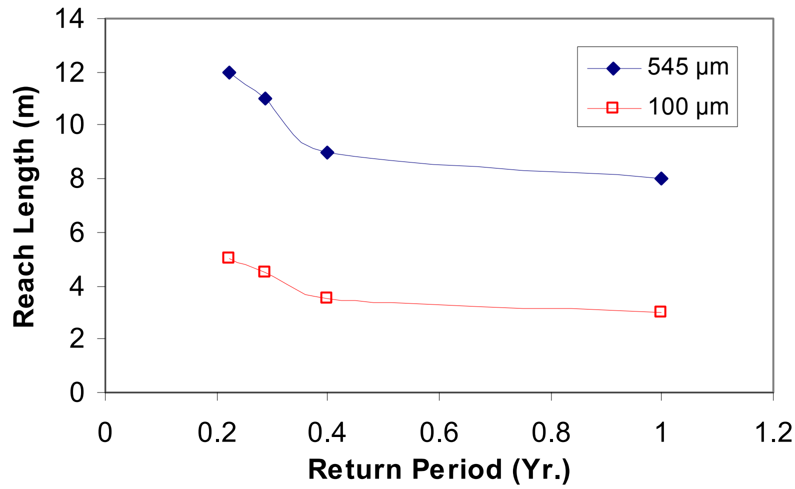

Figure 7. Relationship between intensity of a brief storm (10- 20 min) in Melbourne, Au (Vaze and Chiew, 2003) and the reach length at which particle entrainment due to overland shear would occur. The two curves are for two different diameter particles. Critical shear was approximated using Shield's Curve and flow depths were estimated using Manning's Equation with slope = 0.02 and n = 0.03 (Anderson et al. 1998). Even for smaller particulates (100 µm), overland shear would not play any roll until reach lengths exceeded approximately 4 m.

Conclusions

The primary objective of this work has been to quantify the roles of overland flow and rainfall in particulate wash-off and transport on rough, impervious surfaces when overland shear forces are below a critical threshold for inducing particle movement.

Based on experimental observations, we proposed a model that conceives of particulate ejection and transport on impervious surfaces of moderate slope as a saltation-type process induced by rainfall. This process is similar to so-called 'rain-flow transportation' first noted by Moss et al. (1979) in looking at erosion of soil beds and modeled for soils by Hairsine and Rose (1991).

Overall, the model begins to connect wash-off theory into the much more extensive body of research on soil erosion. However, two features distinguish wash-off from soil erosion: the process quickly becomes sediment limited and the roughness features do not simply affect hydraulic flow characteristics but also the availability of particles. Additional investigation is needed into these two realms to more fully develop the physical wash-off mechanisms.

APPENDIX

A. Experimental Data

Time	Upslope Flow (mL/min) and Rainfall Rate (cm/hr) (150 / 4.9)	(144 / 6.9)	(160 / 8.6)	(148 / 10.9)
0	0.01	0.231	0.196	0.119
1	0	0.091	0.028	0.197
2	0	0.013	0.028	0.047
3	0	0.007	0.028	0.03
4	0.013	0	0.023	0.04
5	0	0	0.023	0.08
6	0.003	0.009	0.023	0.179
7	0	0	0.056	0.491
8	0	0.013	0.144	0.54
9	0	0	0.221	0.465
10	0.002	0.016	0.337	0.485
11	0	0	0.425	0.57
12	0	0.037	0.297	0.596
13	0.007	0.135	0.384	0.373
14	0.006	0.135	0.322	0.322
15	0.006	0.14	0.345	0.322
16	0.012	0.216	0.329	0.247
17	0.025	0.124	0.329	0.247
18	0.021	0.258	0.32	0.18
19	0.037	0.168	0.32	0.18
20	0.058	0.147	0.368	0.171
21	0.111	0.269	0.368	0.171
22	0.143	0.177	0.25	0.13
23	0.151	0.177	0.25	0.13
24	0.136	0.175	0.159	0.097
25	0.161	0.175	0.159	0.097
26	0.141	0.184	0.159	0.097
27	0.164	0.184	0.159	0.097
Add.	0.389	0.85	---	---
Rinse	7.029	5.297	3.348	2.217
Total	8.625	9.228	9.398	8.917

Time	Upslope Flow (mL/min) and Rainfall Rate (cm/hr)				
	(320 / 4.6)	(310 / 8.4)	(310 / 9.0)	(305 / 8.0)	(315 / 11.1)
0	.264	0.145	0.137	0.255	.201
1	0.083	0.012	0.02	0	0.047
2	0.006	0.018	0.02	0.029	0.014
3	0.002	0.018	0.067	0.029	0.427
4	0	0.246	0.621	1.014	1.717
5	0.005	0.991	1.478	1.312	1.277
6	0	1.316	1.447	1.332	1.161
7	0.094	1.435	0.764	0.774	0.758
8	0.481	0.744	0.627	0.574	0.518
9	0.65	0.566	0.554	0.568	0.362
10	0.4	0.505	0.428	0.386	0.4
11	0.462	0.556	0.391	0.357	0.276
12	0.591	0.304	0.195	0.326	0.138
13	0.369	0.268	0.226	0.191	0.138
14	0.257	0.188	0.183	0.166	0.14
15	0.238	0.188	0.183	0.166	0.14
16	0.212	0.142	0.159	0.137	0.124
17	0.212	0.142	0.159	0.137	0.124
18	0.3	0.142	0.159	0.137	0.092
19	0.3	0.092	0.093	0.086	0.092
20	0.27	0.092	0.093	0.086	0.092
21	0.27	0.092	0.093	0.086	0.092
22	0.21	0.103	0.077	0.048	0.062
23	0.21				0.062
24	0.175				0.062
25	0.175				0.062
26	0.123				0.062
27	0.123				0.062
Add.	---	---	---	---	---
Rinse	3.084	0.287	0.249	0.268	0.961
Total	9.302	8.592	8.424	8.464	9.462

	Upslope Flow (mL/min) and Rainfall Rate (cm/hr)			
Time	(440 / 2.9)	(415 / 5.0)	(420 / 7.9)	(415 / 12.1)
0	0.371	0.452	0.748	---
1	0.074	*0.01*	0.091	0.282
2	0.003	0.01	0.009	0.191
3	0	0.002	0.035	2.933
4	0.005	0.003	0.467	1.427
5	0.005	0.052	1.88	0.874
6	0.022	0.065	1.818	0.789
7	0.022	0.765	*0.7*	0.401
8	0.21	0.905	0.303	0.264
9	0.21	0.43	0.435	0.298
10	0.235	0.255	0.349	0.235
11	0.235	0.342	0.213	0.197
12	0.381	0.279	0.205	0.146
13	0.211	0.194	0.185	0.133
14	0.279	0.149	0.161	0.096
15	0.339	0.149	0.161	0.096
16	0.339	0.156	0.135	0.095
17	0.184	0.156	0.135	0.095
18	0.184	0.128	0.101	0.0755
19	0.233	0.128	0.101	0.0755
20	0.233	0.12	0.064	0.0485
21	0.17	0.12	0.064	0.0485
22	0.17	0.185	0.072	*0.04*
23	0.102	0.185	0.072	*0.04*
24	0.102	0.104	0.046	*0.032*
25	0.141	0.104	0.046	0.032
26	0.141	0.112	0.057	0.035
27	0.085	0.112	0.057	0.035
Add.	0.753	---	---	---
Rinse	3.318	3.084	1.178	.419
Total	8.757	8.756	9.888	9.433

Note: Sequential duplicate values typical indicate that a measurement was only taken at the time of the last duplicate. Mass loss was assumed to be uniform across the entire interval and the measured loss was distributed accordingly.

Additionally, italicized values indicate an estimated amount, typically due to loss of the sample during massing.

B. Reassessing Screened Rainfall Experiments

As cited in the introduction, Vaze and Chiew (2003) observed that rainfall and runoff play approximately equal roles in inducing wash-off. While the primary limitation of this work as noted in the introduction was a lack of quantitative analysis, more critical assessment of the experiment suggests that the use of screens to act as a control for rainfall may not be valid. We postulate that Vaze and Chiew may not actually have been comparing a condition representative of overland flow alone to a condition of rainfall.

Background

The standard experimental method to distinguish between the role of rainfall and overland flow has been to use fine mesh screens to dissipate the energy of raindrops and control for the effects of rainfall. Most of these experiments have been carried out on soils. Experiments on bare, tilled soils intended to differentiate between soil detachment from rills and the area between rills found that screens reduced drop energy by 76-89% and reduced erosion in area between rills by 90% (Young and Wiersma, 1973). A study of erosion of dirt roads in tropical locales found that screened treatments resulted in lower, but more uniform, losses over time (Ziegler et al., 2000). One of the few experiments primarily interested in impervious surfaces, Vaze and Chiew conducted plot scale experiments on screened and unscreened impervious, concrete surfaces in Australia (2003). TSS pollutographs indicated that rainfall and overland flow together as well as overland flow alone both resulted in particulate losses, but losses due to overland flow alone were approximately half as large. Vaze and Chiew therefore concluded that rainfall and overland flow played equal roles in transporting particulates. However, the use of screens does not entirely eliminate the energy of the raindrop nor change the axis upon which the drop force is applied to the system. A force applied normal to the particle would be likely to more readily eject it than one applied tangentially. While the energy of the raindrop seems essential for detaching particles from cohesive, bare soils, no experiments suggest that loose particles on impervious surfaces require a similar amount of energy for detachment and should follow a similar trend as soils.

Experimental Methods

Experiments were conducted to directly compared the role of rainfall alone, screened rainfall alone, and runoff alone. The same set-up as in previously discussed experiments was used. The three mass transport runs are summarized in Table B1.

Table B1. Summary of flow rates used in assessment of role of overland flow.

Run	Upslope Flow (mL/min)	Rainfall Equivalent Upslope Flow (cm/hr)	Rainfall (cm/hr)	Notes
1	0	0	21	
2	0	0	21	Screened
3	300	21	0	

Hand-sifted sand of diameter 500-590 μm applied to a 20 cm zone 12 cm from the upslope edge of the chute.. Collection and application were identical to the methods stated in the main text.

The screened run covered the flume with two layers of 1-mm mesh metal screen. The two layers were approximately 2cm and 4cm above the flume bed.

Experimental Results and Discussion

Figure B1. plots incremental mass loss versus time for screened rainfall alone, rainfall alone, and overland flow alone. The findings indicate that overland flow alone was unable to move virtually any sediment but that both screened and unscreened rainfall produced substantial wash-off. As with Vaze and Chiew (2003), we witnessed more wash-off without the screen than with the screen, although the relative differences between the two were smaller than Vaze and Chiew found. This is probably simply due to slight differences in the experimental set-ups; in particular, we used only a single, relatively large particle while they used a distribution of sizes.

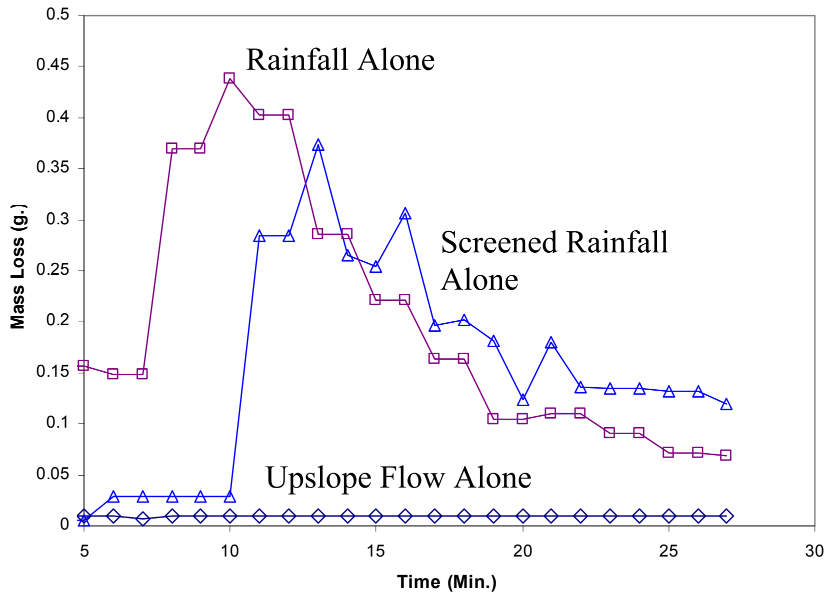

Figure B1. Comparison of sediment loss for equivalent end of flume flow amounts applied by rainfall alone, screened rainfall alone, and by upslope flow alone. When upslope flow alone is applied, minimal mass loss occurs.

The experiment suggests that a distinct difference exists between mass loss due to overland flow alone and mass loss due to screened rainfall. Most critically, screened rainfall is not representative of overland flow, and screened rainfall should not be used as a proxy to elucidate the role of overland flow in dislodging and transporting particles on an impervious surface. In assuming screened rainfall is representative of overland flow, the role of overland flow will be greatly overestimated.

C. Matlab Code

```
%Piece-wise Two-Layer Wash-off Model
%Description:   Quantifies particulate wash-off from a rough, impervious surface. Assumes that particle
%               ejection is caused by raindrops and particle movement is dictated by overland flow.
%               Overland flow does not initiate particle movement.
%               This version differs from TwoLayer in that it uses smaller array sizes and only stores
%               a limited number of intermediate values.
%Date: January 12, 2004
%Written by: Steve Shaw

% ---------------Constants---------------------------------

L=50; % plane length (cm)
W=10; %plane width (cm)
Q=445; %inflow rate (mL/min) % inflow slightly adjusted to account for rain above app transport area
Pq=170; % precip (mL/min)
P=Pq/80/10; %precip rate (cm/min-cm^2)
RoM=.5; %inital particle spatial density in application area (g/cm)
a=130; %drop effectiveness (1/cm) 23
vset=91.8; % settling veloc. (cm/min)  % vset=91.8

dt=.00025; % time step (min) for .5 mm flow, .0004 sec settling time (.00025)
T=20; % end time (min)
DT=1; % reporting interval (min)

Time(1)=DT;
QMinP(1)=0;

%-------------Water Balance (dx determination)--------------

q(1)=Q;
i=1;
l=0;

while l<L
   [V(i),D(i)]=Mannings(q(i));
   dx(i)=V(i)*dt;
   l=l+dx(i);
   q(i+1)=dx(i).*P.*W+q(i);
   i=i+1;

end %while

TotalCells=i-1; %sets total number of cells in system

%---------Create Arrays-------------------------------------------


h=zeros(TotalCells,(DT/dt));
e=zeros(TotalCells,(DT/dt));
Mg=zeros(TotalCells,(DT/dt+1));
Ms=zeros(TotalCells,(DT/dt+1));
```

```
%----------Initial Mass Distribution-------------------------------------

i=1;
appL=0; %application zone length counter
MTot=0;

while appL<20 % where 20 is application zone length
   Mg(i,1)=dx(i)*RoM;
   appL=dx(i)+appL;
   MTot=MTot+Mg(i,1); % a check on mass amount applied - should be approx. 10 g
   i=i+1;
end %while

AppCells=i; % sets point at which cells have no intial application

j=0;
for j=AppCells:TotalCells
   Mg(j,1)=0;
end % for


%-----------Initial Suspended Mass Distribution (t=1)--------------

x=0;
for x=1:TotalCells
   Ms(x,1)=0;
end % for


%---------at time=1 (moment after first raindrop impact)-----

x=0;
for x=1:TotalCells
   h(x,1)=0;
   e(x,1)=a*P*SurfaceMAvail(Mg(x,1),dx(x));
end; % for

Ms(1,2)=.5*e(1,1)*dt;
Mg(1,2)=-Ms(1,2)+Mg(1,1);

x=0;
for x=2:TotalCells
   Ms(x,2)=.5*e(x,1)*dt+.5*e(x-1,1)*dt+Ms(x-1,1);
   Mg(x,2)=-Ms(x,2)+Mg(x,1); % where Ms(:,1)=0
   if Mg(x,2)<0
      Mg(x,2)=0;
   end %if
end %for


%----Establish first set of initial values--------------------------

InitMs=Ms(:,2);
InitMg=Mg(:,2);
Inith=h(:,1);
```

```
Inite=e(:,1);
InitMgCum=MTot;

%-----start interval calculations

i=1;
for i=1:T/DT

  i

  %------at x=1 for all t>1------------------------------------------

  for t=2:(DT/dt+1)

    Ms(:,1)=InitMs;
    Mg(:,1)=InitMg;

  h(1,t)=vset*Ms(1,t-1)/D(1);
  if (h(1,t)*dt)>Ms(1,t-1)
    h(1,t)=Ms(1,t-1)/dt;
  end % if/else

  e(1,t)=a*P*SurfaceMAvail(Mg(1,t-1),dx(1));
  if (e(1,t)*dt)>Mg(1,t-1)
    e(1,t)=Mg(1,t-1)/dt;
  end % if/else

  Ms(1,t)=(.5*e(1,t)-.5*h(1,t))*dt+Ms(1,t-1);
  Mg(1,t)=-Ms(1,t)+Mg(1,t-1);

  end %for

  %------for x>1 for all t>1----------------------------------------

  for t=2:(DT/dt+1)

    Ms(:,1)=InitMs;
    Mg(:,1)=InitMg;

    for x=2:TotalCells

      h(x,t)=vset*Ms(x,t-1)/D(x);
      if (h(x,t)*dt)>Ms(x,t-1)
        h(x,t)=Ms(x,t-1)/dt;
      end % if/else

      e(x,t)=a*P*SurfaceMAvail(Mg(x,t-1),dx(x));
      if (e(x,t)*dt)>Mg(x,t-1)
        e(x,t)=Mg(x,t-1)/dt;
      end % if/else

      Ms(x,t)=(.5*e(x,t)+.5*e(x-1,t))*dt-(.5*h(x,t)+.5*h(x-1,t))*dt+Ms(x-1,t-1);
      Mg(x,t)=-Ms(x,t)+Ms(x-1,t-1)+Mg(x,t-1); % should this be Mg(x-1,t-1)?

    end % x for
```

```
      MLoss(1)=0;
      MLoss(t)=Ms(TotalCells,t)+.5*e(TotalCells,t)*dt-.5*h(TotalCells,t)*dt+MLoss(t-1);

    end % t for

    QMinP(i)=MLoss((DT/dt)+1)*(1/DT); % calcs. cumulative loss during interval
    Time(i)=i*DT;  % in min.

%------End Interval
InitMs=Ms(:,DT/dt);
InitMg=Mg(:,DT/dt);
%InitMgCum=MgCum(DT/dt);

end %for


Obs_Time1=[1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27];
%Obs_Time1=[.5 1.5 2.5 3.5 4.5 5.5 6.5 7.5 8.5 9.5 10.5 11.5 12.5 13.5 14.5 15.5 16.5 17.5 18.5 19.5
20.5 21.5 22 23 24 25 26 ];
plot(Time,QMinP);
axis([0 20 0 3]);
hold on
plot(Obs_Time1,Observed_415_170,'s');
xlabel('Time (Min.)');
ylabel('Mass Loss (g/min)');
```

Functions called from within M-File:

```
function [Mavail]=SurfaceMAvail(Mg,dx)

%Particle mass loss from a surface is dependent on the amount of mass on the surface.
%The available mass is a fraction of a spatial density Mo, a density at which the addition of
% particles will not change the peak mass loss with all other factors unchanged.

Mo=.5; %(g/cm)

Mavail=Mg^2.0/(Mo*dx)^1.0;



function [V,D]=Mannings(q)

% A function that calculates depth and velocity using Manning's Eqn
% q is in mL/min
% V is in cm/min
% D is in cm

n=.01; %Manning's roughness (from experiments)
S= .04; %plane slope
W= .1; %plane width (m)

%qadj=q/1e6/60;
%D=(( (n*qadj)/W/S^.5 )^.6 )*100;
```

```
%V=q/D/(W*100);

V=(10.1*log(q)-46.5)*60;
D=q/(V*10.5);
```

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